4.3 Operations with Radicals

Simplify the radical.

*Find the biggest perfect square that divides evenly into the radicand.

Perfect Squares: 4, 9, 16, 25, 36, 49, 64, ...

$\rightarrow x^2, x^4, x^6, x^8 \leftarrow$ variables with even exponents are perfect squares.

① $\sqrt{50} = \sqrt{25 \cdot 2} = \boxed{5\sqrt{2}}$

② $\sqrt{96} = \sqrt{16 \cdot 6} = \boxed{4\sqrt{6}}$

or

$\sqrt{96} = \sqrt{(2 \cdot 2)(2 \cdot 2)\, 2 \cdot 3}$ *any two of a kind comes out of the radical

$2 \cdot 2\sqrt{2 \cdot 3} = \boxed{4\sqrt{6}}$

③ $\sqrt{x^9} = \sqrt{x^8 \cdot x^1} = \boxed{x^4\sqrt{x}}$

④ $\sqrt{x^{15}} = \sqrt{x^{14} \cdot x^1} = \sqrt{x^{14}} \cdot \sqrt{x} = \boxed{x^7\sqrt{x}}$

(5) $\sqrt{64x^7} = \sqrt{64x^6 \cdot x} = 8x^3\sqrt{x}$

(6) $\sqrt{48x^3y^6} = \sqrt{\underline{16} \cdot 3 \cdot \underline{x^2} \cdot x \cdot \underline{y^6}} = \underline{4xy^3}\sqrt{3x}$

(7) $\sqrt{24x^5y^{11}} = \sqrt{4 \cdot 6 \cdot x^4 x \cdot y^{10} \cdot y} = 2x^2y^5\sqrt{6xy}$

Multiply Radicals

(8) $(4\sqrt{3})(7\sqrt{6})$

$4 \cdot 7\sqrt{3 \cdot 6}$

$28\sqrt{18}$

$28\sqrt{9 \cdot 2} = 28 \cdot 3\sqrt{2}$

$\boxed{84\sqrt{2}}$

* must be same root

* radicand doesn't have to be the same.

(9) $(2\sqrt{15})(-3\sqrt{40}) = -6\sqrt{600}$

$= -6\sqrt{100 \cdot 6}$

$= -6 \cdot 10\sqrt{6} = \boxed{-60\sqrt{6}}$

Add and Subtract Radicals

(10) $\underline{4\sqrt{5}} + 2\sqrt{3} \underline{- 2\sqrt{5}}$

$(4-2)\sqrt{5} + 2\sqrt{3}$

$\boxed{2\sqrt{5} + 2\sqrt{3}}$

* must have same root and radicand.

(11) $4\sqrt{48} + 2\sqrt{125} - 6\sqrt{27}$

$4\sqrt{16\cdot 3} + 2\sqrt{25\cdot 5} - 6\sqrt{9\cdot 3}$

$4\cdot 4\sqrt{3} + 2\cdot 5\sqrt{5} - 6\cdot 3\sqrt{3}$

$16\sqrt{3} + 10\sqrt{5} - 18\sqrt{3}$

$\boxed{-2\sqrt{3} + 10\sqrt{5}}$

Simplify Completely

(12) $(5+\sqrt{2})(\sqrt{3}-\sqrt{6})$

$5\sqrt{3} - 5\sqrt{6} + \sqrt{6} - \sqrt{12}$ $\quad \sqrt{4\cdot 3} = 2\sqrt{3}$

$5\sqrt{3} - 4\sqrt{6} - 2\sqrt{3}$

$\boxed{3\sqrt{3} - 4\sqrt{6}}$

(14) $(4\sqrt{2} + 5\sqrt{7})^2 = (4\sqrt{2}+5\sqrt{7})(4\sqrt{2}+5\sqrt{7})$

$16\sqrt{4} + 20\sqrt{14} + 20\sqrt{14} + 25\sqrt{49}$

$16\cdot 2 + 20\sqrt{14} + 20\sqrt{14} + 25\cdot 7$

$32 + 40\sqrt{14} + 175$

$\boxed{207 + 40\sqrt{14}}$

* HINT:

$\sqrt{3}\cdot\sqrt{3} = 3$

$\sqrt{5}\cdot\sqrt{5} = 5$

$\sqrt{x}\cdot\sqrt{x} = x$

(15) $\dfrac{5}{\sqrt{7}}$

can't have a radical in the denominator

$$\frac{5}{\sqrt{7}}\cdot\frac{\sqrt{7}}{\sqrt{7}} = \frac{5\sqrt{7}}{7}$$

(16) $$\frac{12}{\sqrt{2}} = \frac{12\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = \frac{12\sqrt{2}}{2} = \boxed{6\sqrt{2}}$$

(17) $$\frac{\sqrt{5}}{\sqrt{10}} = \frac{\sqrt{5}\cdot\sqrt{10}}{\sqrt{10}\cdot\sqrt{10}} = \frac{\sqrt{50}}{10} = \frac{5\sqrt{2}}{10} = \boxed{\frac{\sqrt{2}}{5}}$$

(18) $$(\sqrt{3}+2)(\sqrt{3}-2)$$

$$3 \underline{-2\sqrt{3}+2\sqrt{3}} -4$$

$$-1 + 0 = \boxed{-1}$$

(19) $$\frac{4}{\sqrt{6}+3} = \frac{4\cdot(\sqrt{6}-3)}{(\sqrt{6}+3)\cdot(\sqrt{6}-3)} = \frac{4\sqrt{6}-12}{6-9}$$

$$= \boxed{\frac{4\sqrt{6}-12}{-3}}$$

or

$$\boxed{\frac{-4\sqrt{6}+12}{3}}$$

(20) $$\frac{\sqrt{7}}{\sqrt{3}-5} = \frac{\sqrt{7}\cdot(\sqrt{3}+5)}{(\sqrt{3}-5)\cdot(\sqrt{3}+5)} = \frac{\sqrt{21}+5\sqrt{7}}{3-25}$$

$$= \frac{\sqrt{21}+5\sqrt{7}}{-22}$$

$$= \boxed{\frac{-\sqrt{21}-5\sqrt{7}}{22}}$$