

What do we know about inverse functions?

- Reflection over $y=x$
- Switch your x and y → This will switch domain and range

Function with Domain and Range	Graph of original function.	Find the Inverse (Switch x and y , and solve for y .)	Graph of Inverse.	Inverse with Domain and Range.																		
<p>1.</p> <p>$y = x^2 - 3$</p> <p>Domain: $(-\infty, \infty)$</p> <p>Range: $[-3, \infty)$</p>	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td>X</td><td>Y</td></tr> <tr><td>-1</td><td>-2</td></tr> <tr><td>0</td><td>-3</td></tr> <tr><td>1</td><td>-2</td></tr> </table>	X	Y	-1	-2	0	-3	1	-2	$X = Y^2 - 3$ $+3 \quad +3$ $X + 3 = Y^2$ $\sqrt{X+3} = \sqrt{Y^2}$ $Y = (\pm)\sqrt{X+3}$	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td>X</td><td>Y</td></tr> <tr><td>-3</td><td>0</td></tr> <tr><td>-2</td><td>1</td></tr> <tr><td>-2</td><td>-1</td></tr> <tr><td>-1</td><td>-2</td></tr> </table>	X	Y	-3	0	-2	1	-2	-1	-1	-2	$Y = \pm\sqrt{X+3}$ <p>Domain: $[-3, \infty)$</p> <p>Range: $(-\infty, \infty)$</p>
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<p>2.</p> <p>$y = (x-2)^2$</p> <p>Domain: $(-\infty, \infty)$</p> <p>Range: $[0, \infty)$</p>	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td>X</td><td>Y</td></tr> <tr><td>1</td><td>1</td></tr> <tr><td>2</td><td>0</td></tr> <tr><td>3</td><td>1</td></tr> </table>	X	Y	1	1	2	0	3	1	$X = (Y-2)^2$ $\sqrt{X} = \sqrt{(Y-2)^2}$ $\pm\sqrt{X} = Y - 2$ $+2 \quad +2$ $Y = \pm\sqrt{X} + 2$	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td>X</td><td>Y</td></tr> <tr><td>0</td><td>2</td></tr> <tr><td>1</td><td>3</td></tr> <tr><td>4</td><td>4</td></tr> </table>	X	Y	0	2	1	3	4	4	$Y = \pm\sqrt{X} + 2$ <p>Domain: $[0, \infty)$</p> <p>Range: $(-\infty, \infty)$</p>		
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<p>5.</p> $y = \sqrt{x-2}$ <p>Domain: $[2, \infty)$</p> <p>Range: $[0, \infty)$</p>	<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr><th>X</th><th>Y</th></tr> </thead> <tbody> <tr><td>2</td><td>0</td></tr> <tr><td>3</td><td>1</td></tr> <tr><td>6</td><td>2</td></tr> </tbody> </table>	X	Y	2	0	3	1	6	2	$X = \sqrt{Y-2}$ $(X)^2 = (\sqrt{Y-2})^2$ $X^2 = Y - 2$ $+ 2$ $Y = X^2 + 2$	<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr><th>X</th><th>Y</th></tr> </thead> <tbody> <tr><td>0</td><td>2</td></tr> <tr><td>1</td><td>3</td></tr> <tr><td>2</td><td>6</td></tr> </tbody> </table>	X	Y	0	2	1	3	2	6	$Y = X^2 + 2;$ $X \geq 0$ <p>Domain: $[0, \infty)$</p> <p>Range: $[2, \infty)$</p> <p style="text-align: right;">SAME</p>
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Composition of Functions

Linear

$$f(x) = 3x + 1$$

$$g(x) = x - 9$$

$$f(g(x)) = 3(x-9) + 1$$

$$= 3x - 27 + 1 = \boxed{3x - 26}$$

$$g(f(x)) = (3x+1) - 9$$

$$= \boxed{3x - 8}$$

Quadratic

$$f(x) = 3x^2 - 5$$

$$g(x) = x + 2$$

$$f(g(x)) = 3(x+2)^2 - 5$$

$$= 3(x^2 + 4x + 4) - 5$$

$$= 3x^2 + 12x + 12 - 5 = \boxed{3x^2 + 12x + 7}$$

$$g(f(x)) = (3x^2 - 5) + 2$$

$$= \boxed{3x^2 - 3}$$

Radical (Square Root)

$$f(x) = \sqrt{x+5}$$

$$g(x) = 2x - 1$$

$$f(g(x)) = \sqrt{(2x-1)+5}$$

$$= \boxed{\sqrt{2x+4}}$$

$$g(f(x)) = 2(\sqrt{x+5}) - 1$$

$$= \boxed{2\sqrt{x+5} - 1}$$

Radical (Square Root)

$$f(x) = \sqrt{x-3}$$

$$g(x) = 4x^2$$

$$f(g(x)) = \sqrt{(4x^2)-3}$$

$$= \boxed{\sqrt{4x^2-3}}$$

$$g(f(x)) = 4(\sqrt{x-3})^2$$

$$= 4(x-3) = \boxed{4x-12}$$