Solving Radical Inequalities

4 steps

① Set radicand ≥ 0 (keep it real!)

② Solve the inequality

③ Find the interval they have in common (you can make a # line to help)

④ Check a point in solution

Ex 1: $\sqrt{2x+4} \leq 4$

STEP 1
$$\begin{cases} 2x+4 \geq 0 \\ 2x \geq -4 \\ x \geq -2 \end{cases}$$

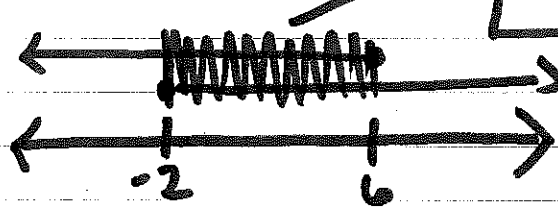
STEP 2
$$\begin{cases} \left(\sqrt{2x+4}\right)^2 \leq (4)^2 \\ 2x+4 \leq 16 \\ 2x \leq 12 \\ x \leq 6 \end{cases}$$

SOLUTION

↓

$[-2, 6]$

STEP 3

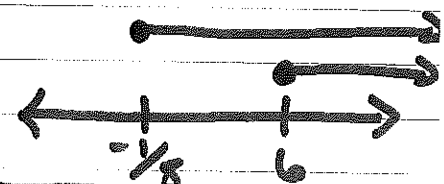


STEP 4
$$\begin{cases} \sqrt{2(0)+4} \leq 4 \\ \sqrt{4} \leq 4 \\ 2 \leq 4 \ \checkmark \end{cases}$$

Ex 2: $\sqrt{8x+1} \geq 7$

$$(\sqrt{8x+1})^2 \geq (7)^2$$
$$8x+1 \geq 49$$
$$8x \geq 48$$
$$x \geq 6$$

$$8x+1 \geq 0$$
$$8x \geq -1$$
$$x \geq -\tfrac{1}{8}$$

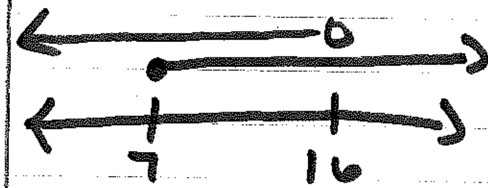


SOLUTION → $\boxed{[6, \infty)}$

You try...

Ex 3: $\sqrt{x-7} + 9 < 12$

$$x-7 \geq 0$$
$$x \geq 7$$



$$\sqrt{x-7} + 9 < 12$$
$$-9 \quad -9$$
$$\sqrt{x-7} < 3$$
$$(\sqrt{x-7})^2 < (3)^2$$
$$x-7 < 9$$
$$x < 16$$

$\boxed{[7, 16)}$