

5.4 Properties of Logarithms Notes

Properties of Logarithms

Property	Algebra	Example
Product	$\log_b(m \cdot n) = \log_b m + \log_b n$	$\log_2(5 \cdot 3) = \log_2(5) + \log_2(3)$ $\log_5(4(x-8)) = \log_5(4) + \log_5(x-8)$
Quotient	$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$	$\log\left(\frac{7}{3}\right) = \log(7) - \log(3)$ $\log_3\left(\frac{4}{x-8}\right) = \log_3(4) - \log_3(x-8)$
Power	$\log_b m^n = n \log_b m$	$\log_4(5^2) = 2 \log_4(5)$ $\log(x-2)^3 = 3 \log(x-2)$

Express as a single logarithm. Simplify, if possible.

1. $\log_3 27 - \log_3 9$

$$\log_3 \frac{27}{9} = \log_3 3$$

2. $5 \log 2$

$$\log 2^5 = \log 32$$

3. $\log_{\frac{1}{3}} x + \log_{\frac{1}{3}} \frac{1}{9}$

$$\log_{\frac{1}{3}} \left(\frac{x}{9}\right)$$

4. $\log_5 2 - 2 \log_5 y$

$$\log_5 \frac{2}{y^2}$$

5. $7 \log_b r + \log_b s - \log_b x$

$$\log_b \left(\frac{r^7 s}{x}\right)$$

Expand the logarithms.

6. $\log_{10} 7x$

$$\log 7 + \log x$$

7. $\log_2 2xy^2$

$$\log_2(2) + \log_2(x) + 2 \log_2(y)$$

8. $\log_7 \frac{x^3}{y^4}$

$$3 \log(x) - 4 \log(y)$$

Solve or Simplify, if possible.

9. $\log_4 4x$

$$\log_4 4 + \log_4 x = 1 + \log_4 x$$

10. $\log_6 \frac{1}{36}$

$$\log_6 1 - \log_6 36$$

$$0 - 2 = -2$$

11. $3^{\log_3 x} = y$

$$x = y$$

12. $2^{3 \log_2 3}$

$$\log_2 x = 3 \log_2 3$$

$$x = 3^3 = 27$$

13. $\log 1000 - \log 10$

$$\log \frac{1000}{10}$$

$$\log 100$$

$$2$$

14. $3 \log_6 2 + 3 \log_6 3$

$$\log_6 8(27)$$

$$\log_6 216$$

$$6^x = 216$$

$$x = 3$$

Given $\log_m a = 7$, $\log_m b = 8$, $\log_m c = 3$, and $\log_m d = 2$, estimate the value of the following logarithms.

15. $\log_m(ab)$ $\log_m a + \log_m b$
 $7 + 8$
 $\boxed{15}$

18. $\log_m\left(\frac{d^3}{c}\right)$ $3 \log_m d - \log_m c$
 $3(2) - 3 = \boxed{3}$

16. $\log_m\left(\frac{a}{bd}\right)$
 $\log_m a - (\log_m b + \log_m d)$
 $7 - (8 + 2) = 7 - 10 = \boxed{-3}$

19. $\log_m\left(\frac{ab}{c}\right)$
 $\log_m a + \log_m b - \log_m c$
 $7 + 8 - 3$
 $\boxed{12}$

17. $\log_m cb^2$

$\log_m c + 2 \log_m b$
 $3 + 2(8)$
 $\boxed{19}$