5.5 Notes – Solving Exponential and Logarithmic Equations

Warm Up:

1. $\log 2x = \log 25 + \log 4$

$\log(2x) = \log(100)$

$\frac{2x}{2} = \frac{100}{2}$

$\boxed{x = 50}$

2. $2\log_5 x = \log_5 12 + \log_5 75$

$\log_5(x^2) = \log_5(900)$

$x^2 = 900$

$\boxed{x = 30}$ ~~$x = -30$~~

The following problems have a log on one side. Solve for x.

1. $\log 50 + \log\frac{x}{2} = 2$

$\log\left(\frac{50x}{2}\right) = 2$

$\log(25x) = 2$

$10^2 = 25x$

$\frac{100}{25} = \frac{25x}{25}$

$\boxed{x = 4}$

2. $\log_3(x-4) + \log_3(x+4) = 2$

$(x-4)(x+4)$

$x^2 + 4x - 4x - 16$

$x^2 - 16$

$\log_3(x^2 - 16) = 2$

$3^2 = x^2 - 16$

$9 = x^2 - 16$

$x^2 = 25$

$\boxed{x = 5}$ ~~$x = -5$~~

3. $\log_{10} x + \log_{10}(x-9) = 1$

$\log_{10}(x^2 - 9x) = 1$

$10^1 = x^2 - 9x$

$0 = x^2 - 9x - 10$

$0 = (x-10)(x+1)$

$\boxed{x = 10}$ ~~$x = -1$~~

4. $\log_6(x-9) + \log_6 x = 2$

$\log_6(x^2 - 9x) = 2$

$6^2 = x^2 - 9x$

$0 = x^2 - 9x - 36$

$0 = (x-12)(x+3)$

$\boxed{x = 12}$ ~~$x = -3$~~

5. $\log_{16}(x^2+2x)=\frac{3}{4}$

$\sqrt[4]{16}^3$ $16^{3/4} = x^2+2x$

$(2)^3$ $8 = x^2+2x$

8 $0 = x^2+2x-8$

$0=(x+4)(x-2)$

$x=-4$ $x=2$

6. $\log_{10} x - \log_{10}(x-3) = -1$

$\log_{10}\left(\frac{x}{x-3}\right) = -1$

$10^{-1} = \frac{x}{x-3}$

$\frac{1}{10} = \frac{x}{x-3}$

$x-3 = 10x$

$\frac{-3}{9} = \frac{9x}{9}$ $x=-\frac{1}{3}$

NO SOL.

7. $\log_7(5x+5)-\log_7(x^2-1)=0$

$\log_7\left(\frac{5x+5}{x^2-1}\right)=0$

$7^0 = \frac{5x+5}{x^2-1}$

$1 = \frac{5x+5}{x^2-1}$

$x^2-1 = 5x+5$

$x^2-5x-6=0$

$(x-6)(x+1)=0$

$x=6$ $x=-1$

log can't be 0

8. $\log_6(b^2+2)+\log_6 2=2$

$\log_6(2b^2+4)=2$

$6^2 = 2b^2+4$

$\frac{32}{2} = \frac{2b^2}{2}$

$\sqrt{16} = \sqrt{b^2}$

$b=\pm 4$

9. $\log_3(x+3)=1-\log_3(x+5)$

$\log_3(x+3)+\log_3(x+5)=1$

$\log_3(x^2+8x+15)=1$

$3^1 = x^2+8x+15$

$0 = x^2+8x+12$

$0=(x+6)(x+2)$

$x=-6$ $x=-2$

$(x+3)(x+5)$
$x^2+5x+3x+15$
$x^2+8x+15$

10. $\log_5(x+5)=1+\log_5(x-3)$

$\log_5(x+5)-\log_5(x-3)=1$

$\log_5\left(\frac{x+5}{x-3}\right)=1$

$5^1 = \frac{x+5}{x-3}$

$5x-15 = x+5$

$4x=20$

$x=5$