PAP Algebra II

5.7 Notes – Base *e* and Natural Logarithms

Name ____________________

Period ________

Base *e* and Natural Logarithms

$A = P(1+\frac{r}{n})^{nt}$ $P=1$ $t=1$

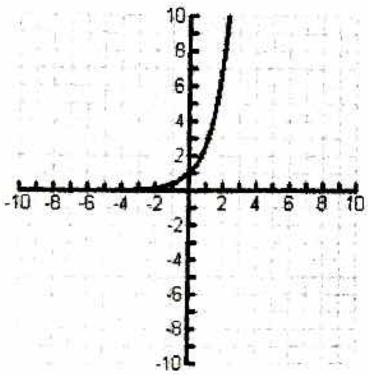
$r = 100\% = 1$

Natural base exponential functions are used extensively in science and mathematics and are used to model quantities that grow and decay continuously. The number *e* is an irrational number whose value is approximately 2.7183.

An exponential function with base *e* is called a natural base exponential function. The equation is written as $y = e^x$.

$n = \begin{cases} 1, 2, 4, 12, 365, \\ 8760, 525600, \\ 31536000 \end{cases}$

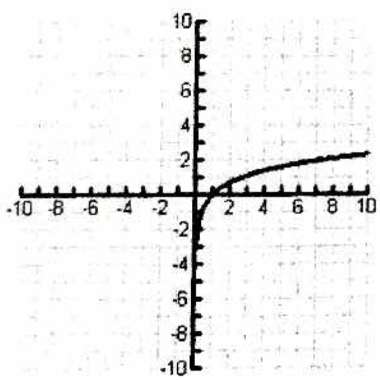
Graph $y = e^x$



	End Behavior
Domain: $\mathbb{R}$ or $(-\infty, \infty)$	$\lim_{x\to+\infty} f(x) = +\infty$
Range: $y > 0$ or $(0, \infty)$	$\lim_{x\to-\infty} f(x) = 0$
Asymptote: $y = 0$	

The logarithm with base *e* is called the natural logarithm. The equation of the natural logarithm is $y = \log_e x$ which is normally written as $y = \ln x$. Therefore $\log_e x = \ln x$. All properties of logarithms that you have learned apply to the natural logarithms as well.

Graph $y = \ln x$



	End Behavior
Domain: $x > 0$ or $(0, \infty)$	$\lim_{x\to+\infty} f(x) = +\infty$
Range: $\mathbb{R}$ or $(-\infty, \infty)$	$\lim_{x\to 0} f(x) = -\infty$
Asymptote: $x = 0$	

Exponential Equation	**Logarithmic Equation**
$y = e^x$	$x = \ln y$

Write an equivalent exponential or logarithmic equation.

1. $\ln 5.2 = x$

   $e^x = 5.2$

2. $\ln 18 = 3x$

   $e^{3x} = 18$

3. $e^x = 5$

   $\ln 5 = x$

4. $e^4 = 12x$

   $\ln(12x) = 4$

Evaluate each equation.

5. $e^{\ln x} = y$

$\ln y = \ln x$

$y = x$

6. $e^{\ln 7} = y$

$\ln y = \ln 7$

$y = 7$

7. $\ln e^{x} = y$

$e^{y} = e^{x}$

$y = x$

8. $\ln e^{4x+3} = y$

$e^{y} = e^{4x+3}$

$y = 4x+3$

$* \; e^{\ln b} = b$

$\ln e^{b} = b$

Properties of Natural Logs

Property	Algebra	Example
Product	$\ln(m \cdot n) = \ln(m) + \ln(n)$	$\ln(5x) = \ln(5) + \ln(x)$
Quotient	$\ln\left(\frac{m}{n}\right) = \ln(m) - \ln(n)$	$\ln\left(\frac{x+2}{x-7}\right) = \ln(x+2) + \ln(x-7)$
Power	$\ln(m^{n}) = n\ln(m)$	$\ln x^{\frac{1}{3}} = \frac{1}{3}\ln x$

Simplify.

9. $4\ln 2$

$\ln 2^{4}$

$\ln 16$

10. $2\ln x$

$\ln x^{2}$

11. $\ln 10 - 5\ln 7$

$\ln 10 - \ln 7^{5}$

$\ln\left(\frac{10}{7^{5}}\right)$

12. $3\ln x + 3\ln y$

$\ln x^{3} + \ln y^{3}$

$\ln(x^{3}y^{3})$

13. $\ln v + 4\ln w + \frac{\ln u}{3}$

$\ln v + \ln w^{4} + \ln u^{1/3}$

$\ln(v w^{4} u^{1/3})$

14. $4\ln u - \ln w - 2\ln v$

$\ln u^{4} - \ln w - \ln v^{2}$

$\ln\left(\frac{u^{4}}{wv^{2}}\right)$

Solve each equation.

15. $\ln 4x = 3$

$e^{3} = 4x$

$\boxed{x = \frac{e^{3}}{4}}$

16. $\ln(x-6) = 1$

$e^{1} = x-6$

$\boxed{x = e+6}$

17. $\ln 8 + \ln x = 1.6$

$\ln(8x) = 1.6$

$e^{1.6} = 8x$

$\boxed{x = \frac{e^{1.6}}{8}}$

18. $\ln 3x + \ln 2x = 9$

$\ln 6x^2 = 9$

$e^9 = 6x^2$

$x^2 = \frac{e^9}{6}$

$\boxed{x = \sqrt{\frac{e^9}{6}}}$

19. $\ln 5x + \ln x = 7$

$\ln 5x^2 = 7$

$e^7 = 5x^2$

$x^2 = \frac{e^7}{5}$

$\boxed{x = \sqrt{\frac{e^7}{5}}}$

20. $e^{4x} = 120$

$\ln 120 = 4x$

$\boxed{x = \frac{\ln 120}{4}}$

21. $\ln(x+3) - 5 = -2$

$\ln(x+3) = 3$

$e^3 = x+3$

$\boxed{x = e^3 - 3}$

22. $e^{4x-1} - 3 = 12$

$e^{4x-1} = 15$

$\ln 15 = 4x - 1$

$\ln 15 + 1 = 4x$

$\boxed{x = \frac{\ln 15 + 1}{4}}$

$x = \frac{\ln 15}{4} + \frac{1}{4}$

either answer

23. $\ln(5x+3) = 3.6$

$e^{3.6} = 5x+3$

$e^{3.6} - 3 = 5x$

$\boxed{x = \frac{e^{3.6} - 3}{5}}$

or

$\boxed{x = \frac{e^{3.6}}{5} - \frac{3}{5}}$

Compound Interest

The formula for compounded interest is $A = P(1+\frac{r}{n})^{nt}$, where P is the principal amount, r is the annual interest rate as a decimal, n is the number of times it is compounded per year and t is the time in years.

As $n \to +\infty$, $A \to e$. Therefore, the formula for continuously compounded interest is written as $A = Pe^{rt}$, where P is the principal amount, r is the annual interest rate as a decimal, and t is the time in years.

16. Suppose you deposit \$1000 in an account paying 2.5% annual interest, compounding continuously. What is the balance after 10 years?

$P = 1000 \quad r = .025 \quad t = 10$

$A = 1000e^{(.025)(10)}$

$\boxed{A = \$1284.03}$

17. How long will it take for the balance in your account to reach at least \$1500?

$P = 1000 \quad r = .025 \quad A = 1500$

$\frac{1500}{1000} = \frac{1000e^{.025t}}{1000}$

$1.5 = e^{.025t}$

$\ln 1.5 = \ln e^{.025t}$

$.025t = \ln 1.5$

$t = \frac{\ln 1.5}{.025} = 16.219$

At the beginning of year 17 you will have more than \$1500